Reg. No. :

Question Paper Code : 90333

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Second Semester

Civil Engineering

MA 8251 : ENGINEERING MATHEMATICS – II

[Common to all branches (Except Marine Engineering)]

(Regulations 2017)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. Prove that any square matrix A and its transpose have the same eigen values.

2. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, then find $2A^2 - 8A - 10I$, where I is the unit matrix.

3. In what direction from (2, 1, –1) is the directional derivative of $\phi(x, y, z) = x^2 y^2 z^4$ maximum ?

4. State Green's theorem in a plane.

5. Show that the function $f(z) = z\bar{z}$ is nowhere analytic.

6. Find the fixed point(s) of the bilinear transformation $w = \frac{1}{z}$.

7. Evaluate the integral $\int \frac{1}{z^2} dz$ over the entire complex plane.

8. Identify the type of the singularity for the function $f(z) = \frac{\cos z}{z}$ at the point $z = 0$,

9. Find L[t sin at].

10. State sufficiency conditions for the existence of Laplace transform.

PART – B

(5×16=80 Marks)

11. a) i) Find the eigen values and the eigen vectors of the matrix $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ **(8)**

ii) Using Cayley-Hamilton theorem find A^{-1}, if $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$. **(8)**

(OR)

b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Hence find its nature. **(16)**

12. a) i) Verify Gauss divergence theorem for the vector function $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$. **(10)**

ii) Prove that $\vec{F} = yz^2\vec{i} + (xz^2 - 1)\vec{j} + (2xyz - 2)\vec{k}$ is irrotational. Hence find its scalar potential. **(6)**

(OR)

b) i) Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$, in the rectangular region of the $z = 0$ plane bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$. **(10)**

ii) If $\vec{A} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$, evaluate $\int_C \vec{A}.d\vec{r}$, where C is the curve $y = x^2$ in the xy-plane from the point (1, 1) to (2, 4). **(6)**

13. a) i) Determine the analytic function $f(z) = u + iv$, if $u = \dfrac{\sin 2x}{\cosh 2y + \cos 2x}$. **(8)**

ii) Find the bilinear map which maps the points $z = 1, -1, \infty$ in the z-plane onto the points $w = -1, -i, i$ in the w-plane. **(8)**

(OR)

b) i) If $f(z) = u + iv$ is an analytic function, then prove that both u and v are harmonic functions. Justify your answer about the converse of the statement. **(8)**

ii) Find the image of $|z - 2i| = 2$ under the transformation $w = \dfrac{1}{z}$. **(8)**

14. a) i) If $F(a) = \oint_C \frac{(3z^2 + 7z + 1)}{z - a} dz$, where C is $|z| = 2$, then find the values of $F(4)$, $F(1 - i)$ and $F'(1 - i)$. (8)

ii) Using the contour integration, evaluate $\int_0^\infty \frac{dx}{(x^2 + 1)^2}$. (8)

(OR)

b) i) Obtain the Laurent's series expansion of $f(z) = \frac{z^2 - 1}{(z + 2)(z + 3)}$ in the regions $|z| < 2$ and $2 < |z| < 3$. (8)

ii) Evaluate by using contour integration $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$. (8)

15. a) i) Find the Laplace transform of f(t) with period $\frac{2\pi}{\omega}$,

where $f(t) = \begin{cases} \sin\omega t, & \text{for } 0 < t < \frac{\pi}{\omega} \\ 0, & \text{for } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ (8)

ii) Solve by using Laplace transforms $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$, when $y(t) = 1$, $\frac{dy}{dt} = 0$ at $t = 0$. (8)

(OR)

b) i) Evaluate $\int_0^\infty e^{-t}\left(\frac{\cos at - \cos bt}{t}\right) dt$ by using Laplace transform. (8)

ii) Using convolution theorem, find $L^{-1}\left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right]$. (8)